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Evaluating the Bias of Alternative Cost Progress Models: Tests Using Aerospace Industry Acquisition Programs

by

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Captain, United States Marine Corps
B.S., Auburn University, 1986

Submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

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I. INTRODUCTION

A. DEFENSE SPENDING AND PUBLIC INTEREST

In recent years, spending for national defense has increasingly been the focal point of public scrutiny. This trend can be attributed to two underlying causes--the growing federal budget deficit, and the problem of cost overruns in the acquisition of major weapons systems.

1. The Federal Budget Deficit

> Between fiscal years 1980 and 1987, the Department of Defense annual Budget Authority almost doubled, from $143 billion to $281 billion, with its total exceeding $1 trillion during that period. This sharp increase contributed to rising deficits and aroused public concern over the ways defense dollars were being spent. [Ref. 1:p. 8]

The Congressional Budget Office's "Economic and Budget Outlook: Fiscal Years 1992-1996" indicates that since 1986, the defense budget has been on a downward path. Total budget authority[1] for 1991 was down approximately four percent from 1990. Moreover, projections for 1992 and 1993 indicate further decreases. [Ref. 3:p. 84] As a result of these

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[1] Budget authority is "the authority granted to a federal agency in an appropriations bill, to enter into commitments that result in immediate or future spending." Budget authority is not necessarily the amount of money that an agency or department will spend during a fiscal year. Instead, it is merely the upper limit on the amount of new spending commitments it can make. [Ref. 2:p. 176]

trends, it is more important than ever that the Department of Defense (DoD) manages its dollar resources effectively.

One key area in which costs must be managed effectively is the development and procurement of weapons systems[2].

> During much of the past three decades, constant dollar unit costs for major defense systems have grown much faster than constant dollar total budgets for these systems. The result has been the purchase of smaller quantities of new systems, delayed modernization and shrinking capabilities. [Ref. 1:p. 10]

Budget authority and outlays for research, development, test and evaluation and procurement accounted for approximately forty percent of the national defense budget in 1989 [Ref. 4:p. A-146]. More effective and efficient utilization of these funds could result in significant savings within the DoD, thus allowing scarce resources to be applied to other important requirements.

2. Cost Overruns in Weapons Acquisition

The second underlying cause of increased public scrutiny of defense spending is cost overruns. The problem of cost overruns in the weapons acquisition process has been a major source of consternation and embarrassment for the DoD for many years. Numerous researchers and presidential

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[2] The term weapon system normally refers to the major item of equipment and the subsystems, logistical support, software, construction and training needed to operate and support it [Ref. 1:p. 9]. For purposes of this thesis, weapon system refers only to the major item of equipment.

commissions during the past thirty years have concluded that tens of billions of dollars per year could be saved by improving the acquisition process [Ref. 1:p. 32]. "The studies (have) repeatedly urged Congress and the Defense Department to correct five basic deficiencies:

1. Setting requirements for the most sophisticated systems available, often irrespective of cost;
2. Changes in program and contract requirements caused by changes in military user preferences, leading to annual or more frequent changes in program funding levels, initiated by Congress and DoD itself;
3. Lack of incentives for contractors and government personnel to reduce program costs;
4. Failure to develop sufficient numbers of military and civilian personnel with training and experience in business management and in dealing with industrial firms to oversee the development and production of enormous, highly technical industrial programs; and
5. Underestimated schedules and costs of major programs, distorting the decision-making process for the allocation of the national budget." [Ref. 1:p. 32]

While progress has been made in each of the areas identified above, there remains much room for improvement. Major defense procurement programs have repeatedly experienced significant unanticipated schedule delays and cost increases. More than ninety percent of these programs exceed initial cost estimates and, in the majority of cases, the average increase in cost has been more than fifty percent, excluding the effects of quantity changes and inflation. [Ref. 1:p. 32-33]

A cost overrun occurs when the actual cost of a program exceeds the estimated cost. Cost overruns typically occur when: fair initial cost estimates are made but subsequent actual costs are poorly managed and controlled; or actual costs are well-managed but initial cost estimates were unrealistic. [Ref. 5:p. 1]

There are various reasons why initial cost estimates may be unrealistic, particularly unrealistically low. For example, institutional incentives may exist both within DoD and at government contractors to underestimate costs initially in order to get a program approved and started. A second possible reason is that processes, techniques or tools for creating cost estimates may be weak and provide misleading cost forecasts. This thesis addresses a technical question related to the latter issue. In particular, this paper examines the problem of low/unrealistic cost estimates for major weapons programs by analyzing the performance of alternative cost estimation models.

B. COST ESTIMATION MODELS

Broadly, cost estimation models fall into two categories. First, Cost Estimating Relationships (CERs) attempt to explain or predict the cost of a "standard" unit of an item to be manufactured or procured in terms of variables reflecting qualities, attributes or characteristics of the item. For example, aircraft costs may be modeled in terms of speed or

payload. Second, Cost Progress Models attempt to explain or predict changes in unit cost of items over the life of a production or procurement program in terms of changes in the circumstances surrounding production or acquisition. For example, costs may be expected to depend on the numbers acquired and the production rate and thus unit costs may be modeled in terms of such variables. The analysis in this thesis will focus on the latter type of model, cost progress models.

1. The Learning Curve

The most commonly used cost progress model is the learning curve.

> Learning curves have gained widespread acceptance as a tool for planning, analyzing, explaining and predicting the behavior of the unit cost of items produced from a repetitive production process [Ref. 5:p. 1].

Although learning curves were originally developed and applied to predict cost and time requirements for the construction of ships and aircraft during World War II, they have since been applied in many other manufacturing and non-manufacturing settings. The learning curve phenomena was first reported by T. P. Wright in the Journal of Aeronautical Sciences in 1936. Wright observed that, as the quantity of units manufactured doubles, the number of direct labor hours/cost associated with the production of an individual unit decreases at a uniform rate. Moreover, the uniform rate of learning is peculiar to

the manufacturing process being observed.

2. Alternative Cost Progress Models

It is generally acknowledged that other factors, in addition to cumulative quantity, influence unit cost and that the simple learning curve does not provide a fully adequate description of cost behavior. As a result, prior research has attempted to improve the simple learning curve model by including additional variables. [Ref. 5:p. 2]

There are now multiple approaches and models available for estimating the costs of acquisition programs. Two of the most commonly used cost progress model types are the learning curve and the production rate adjustment model. Other model types include the plateau model, the Stanford-B model, the De Jong model and the S-model. These models are differentiated by the variables included and the underlying assumptions.

C. PURPOSE

It is unclear at present which model type is most appropriate for predicting costs under various manufacturing conditions. The purpose of this thesis is to evaluate the quality of cost estimates produced by each of four cost progress models--a random walk model, a learning curve model, a production rate model, and a model incorporating both learning curve and production rate effects. In conducting this evaluation, emphasis will be place on assessing the level of bias associated with each of these models.

D. RESEARCH QUESTIONS

This thesis will address the following research questions:

1. Primary Research Question

What is the bias exhibited by available cost progress models when predicting the future unit cost of weapons systems acquired through a continuing acquisition program?

2. Subsidiary Research Questions

a. Are the various available cost progress models comparable in terms of bias?

b. Do particular models result in less biased estimates under certain conditions?

c. What are those conditions that affect the performance of the models?

d. Can guidelines be established for determining when (under what conditions or circumstances) it is most appropriate to use a particular model type?

E. SCOPE, LIMITATIONS AND ASSUMPTIONS

There are a number of different criteria which can be examined in order to assess various aspects of model performance. Two of these criteria which are particularly important are the accuracy and the level of bias associated with a particular model. Accuracy refers to the degree of error in a model's prediction, without regard to the direction of the error. Bias refers to both the direction and the magnitude of error. It indicates whether predictions made using a particular model underestimate or overestimate actual cost. The focus of this thesis will be limited to an analysis

of the bias associated with the various models tested. One purpose of the study is to either confirm or disconfirm the results of an earlier simulation study by Moses (Learning Curve and Rate Adjustment Models: An Investigation of Bias) and to determine whether or not those results hold when testing real world data. A second purpose is to extend the analysis of bias to a larger set of models.

F. ORGANIZATION OF THESIS

The remainder of this thesis is organized into four chapters. Chapter II provides a review of the literature dealing with various cost progress models. Chapter III provides a description of the sample, data and measures used to conduct the study. Chapter IV provides an analysis of the results. Finally, chapter V summarizes the research findings and suggests directions for future research.

II. REVIEW OF PREVIOUS RESEARCH

Although the progress or learning curve technique of cost estimation was discovered prior to World War II, several decades passed before statistical studies of this phenomena could be readily conducted. This situation can be attributed to two underlying problems: (1) the available data were frequently too sparse to support statistical analyses, and; (2) the sheer volume of calculations and lack of powerful computers meant that many laborious hours were required to perform operations which can today be performed in a few minutes. This second problem had a strong inhibiting effect on researchers. As a result, it was not until the 1950's that significant statistical studies of the learning curve phenomena began to be undertaken. [Ref. 6:p. 8]

This chapter reviews the literature dealing with development of the learning curve and alternative rate adjustment models. In particular, it summarizes the findings of some of the major studies which have been conducted in an effort to evaluate the relative performance of these models. The discussion is organized chronologically into the following groupings: research prior to 1970, research during the seventies, research during the eighties, and research during the nineties.

A. SURVEY OF RESEARCH PRIOR TO 1970

1. Hirsch

In 1952, Hirsch [Ref. 7] published the results of a five-year study of a large United States machine builder. The purpose of the study was to examine the relationship between labor requirements and production volume. Lot size was used as a measure of the rate of production based on the existence of stable production lot intervals. In addition, practically no changes in management or plant and equipment occurred during the period of study. Based on the results of his study, Hirsch concluded the relation between direct labor requirements and lot size was of little consequence in the machining and assembling processes.

2. Cochran

In 1960, E. B. Cochran [Ref. 8] published an article in which he conducted a careful examination of the basic cost function--the learning curve--by studying specific manufacturing conditions and parameters which relate them to cost trends. The purpose of this study was to probe various learning curve applications in an effort to develop new concepts of learning curve analysis and revised applications. Based on the results of his study, Cochran proposed refinements to the basic learning curve concept. In particular, he suggested that learning may not necessarily occur exactly once per unit but instead, may occur either

faster or slower. For example, when planning airframe costs for a four-engine aircraft, it may be appropriate to consider each engine pod as a unit of learning rather than simply consider the aircraft as a unit of learning.

Cochran provided guidelines for accurately identifying the unit of learning and identified other factors, in addition to worker learning, that would affect the rate of cost reduction (e.g., tooling, supervision, parts design and shortages). Moreover, he demonstrated how changing rates of learning and task changes may result in both shifts of the learning curve and non-linear learning. One of the most significant findings presented by Cochran was that:

> Any change in learning rate is equivalent to a change in the unit at which standard cost is reached. And this in turn generates a major shift in the entire level of cost [Ref. 8:p. 319].

As a result of this finding, Cochran concluded "that the determination of learning rate is of major significance in forecasting and controlling costs." [Ref. 8:p. 319]

Cochran indicated that the shape of the learning curve can be critical in the first 100 units and pointed out the fact that there is a wide range of error in straight line curves. Accordingly, he suggested that an S-curve pattern may be more appropriate than the usual linear learning curve.

3. Alchian

In 1963, Alchian [Ref. 9] published the results of a 1949 study conducted for the RAND Corporation. The purpose of this study was to examine the similarity of airframe manufacturing progress functions among various airframe manufacturers. Statistical tests of the similarity of the functions among various airframe manufacturers were performed using World War II data. In addition, the reliability of predictions made with these curves was assessed.

The results of this study indicated the progress functions differed among various airframe types and manufacturing facilities both in the amount and rate of change of required direct labor per pound of airframe. Alchian suggested that, for practical purposes, the use of an average of individual progress functions may be appropriate. By applying this procedure to 22 airframes produced at different facilities, the average production error[3] was found to be

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[3] Production error was defined as follows:

$$\textit{Production Error} = \frac{\textit{Predicted Manhours - Actual Manhours}}{\textit{Actual Manhours}}$$

Direct labor requirements (manhours) for the first 1000 planes were predicted for 22 aircraft model--facility combinations using both an industry progress curve and an airframe type progress curve. The percentage error resulting from the use of each of these curves was then computed for each model--facility combination using the equation described above. Next, the weighted average error per facility (weighted by actual manhours) associated with the use of the industry and airframe progress curves was computed for each of the four major aircraft model groups examined: bombers, fighters, trainers and transports. Based on these figures, the weighted average error per facility for all facilities was computed.

approximately 25 percent. This same result was obtained for the entire output of any particular airframe produced in one facility. Specific curves fitted to the past performance of a particular manufacturing facility resulted in margins of error of approximately 20 percent.

Alchian examined alternative relationships between direct labor per pound of airframe, cumulative number of airframes, time and rate of production. "The results cast doubts on any of the alternatives being better fits than the usual progress curve." [Ref. 9:p. 692]

B. SURVEY OF RESEARCH DURING THE SEVENTIES

1. Linder and Wilbourn

Models which relate costs to various cost-driving features or parameters (i.e., physical/performance parameters of the weapons system) typically result in reasonable estimates of future recurring unit procurement costs [Ref. 10:p. 277]. Nevertheless, Linder and Wilbourn [Ref. 10] suggested that in addition to these parameters, various characteristics of the procurement program itself (e.g., competitive versus sole-source procurement, single-year versus multiple-year buys, and low versus high production rates) represent cost-driving features which should be accounted for in the cost estimating procedure.

Linder and Wilbourn investigated the effect of production rate on recurring missile unit procurement costs. In particular, they developed two models to examine how production rate influences the position and/or slope of the recurring missile hardware cost improvement curve. The first model formulated unit recurring cost as a function of a constant "annual" production rate. The second model formulated unit recurring cost as a function of a variable annualized production rate.

These models were developed based on an analysis of the impact of production rate changes on direct and indirect costs. Linder and Wilbourn reasoned that higher production rates would result in lower fixed costs per unit. Moreover, high production rates were expected to lead to a smaller percentage increase in indirect costs than direct costs. As a result, lower overhead rates should be applied to direct costs. Based on the combined effects of lower fixed costs per unit and lower applied overhead rates, Linder and Wilbourn concluded that unit costs would be reduced at higher production rates, ceteris paribus. Moreover, the cost improvement curve associated with a high production rate was expected to lie below a cost improvement curve associated with a lower production rate.

In assessing the expected effect of production rate on the cost improvement curve slope, the researchers made the following observations:

1. At high production rates, average fixed direct costs are reduced and variable direct costs constitute a larger portion of total direct costs.

2. As direct costs fall, indirect costs are reduced at a slower rate such that overhead rates increase.

These two phenomena have opposite affects on the slope of the cost improvement curve. The first one will tend to increase the slope of the curve due to the influences of learning and other related effects on variable directs costs at a high production rate. The latter phenomena will tend to flatten the curve as production increases. As a result, "the net effect on the cost improvement curve slope depends on the relative amounts of direct and indirect costs per unit as well as the proportion of each cost category which can be considered as fixed or variable." [Ref. 10:p. 280]

Based on the results of their analyses, Linder and Wilbourn reached the following conclusions:

1. Ceteris paribus, higher production rates result in lower unit recurring costs at each production quantity.

2. Doubling the production rate lowers average unit costs by approximately three to seven percent for the quantities examined.

3. Changing the production rate has only a slight influence on the slope of the unit cost improvement curve.

4. The effects discussed above are relatively insensitive to changes in the models' parameter values. [Ref. 10:p. 300]

2. RAND Studies

During the 1970's, the RAND Corporation conducted a number of studies which examined the relationship between production rate and airframe costs. The specific objectives and results of two of these studies are summarized below.

a. Large, Hoffmayer and Kontrovich

In 1974, the results of a study by Large, Hoffmayer and Kontrovich [Ref. 11] were published. "The purpose of this study was to investigate the nature, magnitude and causes of the influence of production rate on unit cost." [Ref. 11:p. iii] Based on the assumption that production rate and unit cost vary inversely, the researchers sought to develop an estimating model to express the relationship for various elements of cost.

The results of the analysis suggested the effect of production rate on manufacturing labor, manufacturing materials, tools and engineering could not be predicted with confidence. In any specific case, the effect depended on a number of factors including how rate changes were achieved, the availability of suppliers, the local labor supply, management policy, the timing of rate changes, plant capacity, and plant backlog. The only element of cost which was found to clearly be a function of production rate was overhead.

Based on their findings, the researchers concluded that the influence of production rate on aircraft cost could

not be predicted with any degree of confidence. Each case should be examined separately and in detail to assess the effect of rate. In addition, they suggested that in advanced planning studies, rate effects in aircraft production programs can be ignored because they are far outweighed by other uncertainties. As a result, they indicated that a model that does not explicitly consider rate may be preferable for advance planning purposes.

b. Large, Campbell and Cates

This study, published in 1976, attempted to derive improved parametric equations for estimating the acquisition cost of aircraft airframes. Earlier RAND studies had indicated variations in cost among different airframes were best explained by the quantity produced and aircraft characteristics, i.e., airframe unit weight and maximum speed. Large, Campbell and Cates [Ref. 12] were unable to identify additional characteristics that would make an estimating model more flexible and, hence, better able to deal with characteristics peculiar to individual aircraft. None of the independent variables considered significantly improved the reliability of estimates obtained using only weight, speed and cumulative quantity. As a result, they suggested that future research which examines the influence of program characteristics on program cost may be more productive.

3. Smith

The purpose of this study was to develop and test a procedure to assess the effect of production rate changes on the direct labor requirements for production of additional airframes. Smith [Ref. 13] proposed a cost model to express direct labor hours required as a function of cumulative production and production rate. Two approaches to expressing the production rate variable were examined: a lot average delivery rate, and a lot average manufacturing rate. Data from three airframe production programs--the F-4 program, the F-102 program, and the KC-135 program--were used to construct data sets. These data sets were then examined in the cumulative production and production rate cost model using regression analysis.

Production rate was found to be an important factor in the cost of airframe production, although its effect was subordinate to that of cumulative production. The study demonstrated empirically that production rate can be an important predictor of variation in unit direct labor requirements. In addition, the results suggested an increase in rate up to plant capacity can lead to a decrease in unit labor requirements.

C. SURVEY OF RESEARCH DURING THE EIGHTIES

1. Crouch

The purpose of this study was to investigate possible sources of bias in the slopes of progress functions which are conventionally estimated using the unit cost progress function. Crouch [Ref. 14] asserted that given the fact that unit costs are a function not only of cumulative output but also of the rate of output per time period, the conventional unit-cost progress function has omitted variables. Under these circumstances, the use of ordinary least squares regression may introduce specification bias into the estimates. Crouch confirmed this situation mathematically.

> When a variable from the true relation is omitted, a part of its influence in explaining the movements of the dependent variable is captured by the independent variables which are included. When the omitted variable is not correlated with any of the independent variables, the coefficients of the included variables are not biased. [Ref. 14:p. 42]

Crouch conducted a pilot study to investigate the existence of bias when progress functions are estimated in the conventional manner. Unit-cost data (on an annual average basis) in constant dollars for ten components of the Hawk missile were used. The results of this study indicated that when progress functions are estimated in the conventional manner, biased estimates of the slopes may be obtained in a significant number of cases. In particular, the results

indicated that when returns to scale are constant, the estimated learning curve exponent obtained from the conventional progress function will be unbiased. However, when the returns to scale are not constant, the estimated learning curve exponent obtained from the conventional progress function will be biased. Negative bias will occur when returns to scale are increasing; positive bias will occur when returns to scale are decreasing.

2. Smith

The objective of this study was to examine the impact of production rate changes on the unit cost of weapons systems. Smith [Ref. 15] provided a summary of the significant research on the relationship between production rate and weapon system cost. In doing so, he analyzed the various findings and conclusions and assessed their applicability. The results of his research review indicated that only rather weak conclusions could be drawn from the existing state of knowledge. The principal findings were: production rate affects unit costs but, in most cases, not as strongly as the learning (cumulative quantity) effect, and; the rate effect varies with the weapon system. Smith identified four principal cost-rate models--Womer [Ref. 16], Washburn [Ref. 17], Linder and Wilbourn [Ref. 10], and Fazio and Russell [Ref. 18]. Each of these models differed in the concept of rate, the number of parameters to estimate, and the

range of applicable programs. Nevertheless, none of them was considered suitable for use by top level budget planners. [Ref. 15:p. ii]

Case studies were conducted using production data for six missile systems. These systems possessed a wide range of production characteristics ranging from low volume, labor intensive to high volume, highly automated production. The results of the empirical research supported the belief that under program stretch-out, the most important contributor to increased unit costs is an increased overhead allocation. In addition, the idea that labor inefficiency is often a relatively unimportant factor in rate adjustments was supported. [Ref. 15:pp. 43-44] Based on the results of the case studies, Smith concluded that:

1. A simple rate-sensitive model which focuses on the effect of rate changes on overhead is appropriate for the programming and budgeting phases.
2. Long-range planners should disregard rate behavior and focus only on military requirements. In long-term planning, production rates and their effects are both unpredictable and much less important than other more fundamental considerations. [Ref. 15:p. 46]

3. Balut

Standard use of learning curve theory involves an implicit assumption that overhead is 100 percent variable with direct costs. However, plant overhead is actually comprised of three components--variable overhead, fixed overhead, and

semi-fixed overhead. Variable overhead costs vary with the activity rate. They include production-related indirect costs that are tied to the number of direct laborers working in the plant and the number of units being produced. Fixed overhead costs do not vary with the activity rate and are fixed in the short-term (e.g., depreciation, insurance, rent, security). Semi-fixed overhead is indirect expenses that are partially fixed and partially variable such as utilities. Semi-fixed overhead costs are typically gathered and reported as pools; consequently, the fixed and variable portions are not discernable. [Ref. 19:pp. 63-65]

The costing of alternative aircraft procurement quantities and rates within the Office of the Secretary of Defense (Program Analysis and Evaluation) is a two step process which considers the heterogeneous nature of overhead. First, new average unit prices are derived for each lot, consistent with new lot quantities, using the learning curve. Then, in order to correct for the erroneous underlying assumption in step one, prices are adjusted to reflect the redistribution of fixed overhead resulting from a change in the production rate. This second step is referred to as rate adjustment. [Ref. 19:p. 65]

The objective of Balut's [Ref. 19] study was to evaluate the rate adjustment model used by the Office of the Secretary of Defense by comparing its predictions to actual contractor performance. An improved version of the model

derived using actual contractor data was presented. Balut illustrated the use of this improved model for situations when the contractor has only one program, and for situations when the contractor has other ongoing programs.

4. Bemis

Unit costs for weapons systems have traditionally been projected using the experience curve. This method expresses the projected unit cost as a function of cumulative quantity produced, regardless of the production rate. Prior research into the production rate-cost relationship for weapons systems indicates that unit cost varies significantly as a function of production rate. These variations are, to a great extent, due to the amortization of fixed overhead. [Ref. 20:pp. 84-85]

Bemis [Ref. 20] proposed a method for estimating rate/cost/quantity relationships using system specific cost estimates. Only unit-fly-away costs were considered. The input data for the model was historical rate/cost/quantity data for ongoing programs, and contractor or in-house estimates for new programs. An equation was derived by regressing unit cost on cumulative quantity and production rate. In most of the cases analyzed in this study, a high multiple correlation coefficient (greater than 0.9) was obtained.

Bemis found that when the production rate was stable, the experience curve method and the rate/cost/quantity method

generated identical unit cost estimates. However, when the production rate was variable, lower unit costs were associated with higher production rates, and higher unit costs were associated with lower production rates.

Bemis suggested the rate/cost/quantity model could be an invaluable tool for approaching "what if" questions in the planning and budgeting process. Moreover, he suggested this model offered users a means for readily assessing the cost effects of program stretchouts, the costs of maintaining a warm production base, and the probable effects of program acceleration.

D. SURVEY OF RESEARCH DURING THE NINETIES

1. Boger and Liao

In an effort to reflect the effect of production rate on the cost of weapons systems, researchers have proposed a variety of adjustments to weapons systems cost models. The most popular approach has been to augment the traditional learning curve by adding a rate term. The resulting learning curve is referred to as a rate adjustment model. [Ref. 21:p. 82] Boger and Liao [Ref. 21] examined the effects of different rate measures and cost structures on rate adjustment models and illustrated how alternative surrogate production rate measures might lead to erroneous conclusions.

The effect of production rate on unit cost stems from economies of scale. As production rates are increased,

facilities are utilized more fully and greater specialization of labor occurs. Materials costs are reduced because the increased volume of materials purchased results in quantity discounts. Finally, the increased production volume allows fixed overhead charges to be spread over a larger quantity of output. Together, these underlying effects work to increase efficiency and lower production costs. [Ref. 21:p. 83]

Increases in production rate are normally expected to result in lower unit costs due to economies of scale; however, production rate increases may also lead to diseconomies of scale. Such is the case when a plant is operating beyond its efficient capacity level. Under these circumstances, factors such as over-time pay, lack of skilled labor or the need to invest in additional tooling and/or facilities may lead to inefficiencies and increased unit costs. [Ref. 21:pp. 23-24]

Because of the difficulty associated with measuring production rate, a number of alternative surrogate measures have been adopted. The two primary surrogate production rate measures are lot size and annual/monthly production quantity. Unfortunately, there are weaknesses associated with the use of each of these measures. These weaknesses are as follows:

1. Lot Size: The time required to produce successive, comparably-sized lots frequently changes over the life of a program. As a result, it is unclear what is being measured by the lot size proxy.

2. Production Quantity/Time Interval: If there is a large amount of work-in-progress and the production period is long compared to the observation period, units produced

in the following time period will actually reflect work performed in the previous time period. This can result in substantial bias in estimation.

3. Average Rate for Each Program: This approach may understate the effect of descriptive rate changes. An average rate for each program is usually used in cross-sectional analysis because the production rate may change in a typical production run.

4. Cumulative Quantity: Cumulative quantity is highly correlated with each of the surrogate production rate measures discussed above. As a result, analysts have been unable to separate statistically the effect of learning and production rate. [Ref. 21:pp. 86-87]

In order to avoid some of the difficulties associated with using these surrogate production rate measures, Boger and Liao recommended a ratio of these measures be used. This ratio should be keyed to a base production rate. Adoption of this approach offers a number of advantages:

1. Using a surrogate production rate ratio tends to mitigate the multicollinearity problem.

2. Using the rate to which the manufacturer has tooled as the base rate provides an indicator of returns to variable inputs. Ratios greater than one indicate decreasing returns while ratios less than one indicate increasing returns. [Ref. 21:p. 88]

In addition to examining the problems associated with the two primary surrogate rate measures, Boger and Liao examined the problem of changing cost structures. These changes occur as a result of changes in the production setup. Based on their analysis, the researchers concluded that rate adjustment models are appropriate only when applied to data

collected from plants which have not undergone changes in cost structure.

2. Moses

In 1991, Moses [Ref. 5] published the results of a study in which he investigated and compared the forecasting bias for the learning curve model and a rate adjustment model. Specific objectives of the study were as follows: to determine if either the learning curve model or rate adjustment model exhibits consistent/systematic bias; to determine under what circumstances the two models are biased, and; to identify the nature of the bias (i.e., overestimation or underestimation of future costs).

A simulation approach was used to conduct the research. First, cost series were generated under varying simulated conditions. Then, model parameters were estimated by fitting the learning curve and rate adjustment models to the cost series. Future costs were predicted using each of the models. These predicted future costs were then compared with the actual cost to measure bias. Finally, the relationship between the level of bias and the simulated conditions was investigated using analysis of variance.

The simulation was conducted by varying seven factors which had been found to affect the magnitude of model prediction errors in prior research. These seven factors were:

1. Data History--the number of data points available to estimate the parameters for a model.

2. Variable Cost Learning Rate--the learning curve exponent.

3. Fixed Cost Burden--the proportion of total cost comprised of fixed costs.

4. Production Rate Trend--the production trend during the model estimation period (i.e., gradual growing trend or level trend).

5. Production Rate Instability/Variance--period-to-period fluctuations in the production rate perhaps caused by changes in the demand for output or the supply of inputs, and annual budget uncertainties.

6. Cost Noise Variance--variability in period-to-period cost--designed to reflect unsystematic, unanticipated, non-recurring random factors (e.g., changes in the cost, type or availability of input resources, temporary variations in the level of efficiency, and unplanned changes in the production process).

7. Future Production Level--the production rate planned for the future relative to past levels.

Bias was measured separately for each model as follows:

$$BIAS = \frac{PUC - AUC}{AUC}$$

where

PUC = Predicted unit cost either the learning curve or the rate adjustment model.
AUC = Actual unit cost generated by the cost function.

In conducting his analysis, Moses found that the rate adjustment model provided unbiased cost estimates while the learning curve consistently underestimated actual costs. The

following conclusions regarding learning curve bias were drawn:

1. Learning curve bias stems from the fact that a portion of total cost is fixed. The log linear relationship between cost and quantity assumed by the learning curve does not hold when fixed costs, which are not subject to learning, are present.

2. Bias increases as the proportion of total cost made up of fixed costs increases. This relationship holds up to the point where fixed costs account for 50 percent of total cost; further increases above that level reduce bias.

3. The production rate during the period of model estimation and the production rate during the period for which costs are forecast both affect the degree of bias. Bias is minimized when there is a consistent production trend during these periods. Bias is magnified when there is a shift in production rate trend.

4. The steeper the learning curve slope, the greater the level of bias. (This conclusion is based on the assumption that the proportions of total cost that are fixed or variable remain relatively stable.)

5. The greater the number of observations, the higher degree of bias.

6. The further into the future predictions are made, the greater the degree of bias.

E. SUMMARY

The review of previous research conducted in this chapter, while comprehensive, is by no means all-inclusive. Nevertheless, the studies discussed do provide a sound basis for assessing the current level of understanding with respect to learning curves and rate adjustment models. The following

conclusions can be drawn concerning the research efforts to date:

1. The broad objective of most of the studies dealing with extensions or modifications to the learning curve has been to investigate the relationship between production rate and cost to determine whether or not consideration of production rate leads to improved cost estimates.

2. Findings concerning the importance of considering the effect of production rate on unit costs when predicting weapons systems costs have been inconclusive. Some studies have found production rate to be an important predictor of cost variation while other studies found production rate to be of little or no significance.

3. Production rate affects unit cost but, in most cases, not as strongly as the learning (cumulative quantity) effect.

4. The production rate effect varies with the weapon system and the specific cost elements.

5. The influence of production rate on unit cost depends on the relative amounts of direct and indirect costs per unit and the proportion of each category that can be considered as fixed or variable.

6. Variations in unit cost in response to production rate changes stem from economies of scale. These variations are, to a great extent, due to the amortization of fixed overhead.

7. Lower unit costs are associated with high production rates and higher unit costs are associated with lower production rates.

8. The rate adjustment model, as described by Moses (1991), provides unbiased cost estimates. Conversely, the learning curve consistently underestimates actual costs.

9. Learning curve bias stems from the fact that a portion of total cost is fixed.

10. Learning curve bias is affected by the proportion of total cost made up of fixed costs, the production rate during both the period of model estimation and the period for which costs are forecast, the slope of the learning curve, the number of observations, and the time horizon for which predictions are being made.

Findings by Moses [Ref. 5] concerning the bias associated with the traditional learning curve and the rate adjustment model are noteworthy. Nevertheless, their significance is somewhat tempered by the use of simulated data. The remainder of this thesis will focus on confirming or disconfirming the results of the Moses [Ref. 5] study and will extend the research to include two additional models--a random walk model and a model incorporating production rate effects.

III. SAMPLE, DATA AND VARIABLES

The purpose of this chapter is to provide an overview of the methodology used to conduct this study. The chapter will begin with a description of the population from which the sample was drawn, the criteria for inclusion of a weapon system/procurement program in the sample and the specific raw data collected for each weapon system. This will be followed by a description of the procedure used to "expand" the sample. Next, the four cost progress models that were included in the study will be introduced along with the procedure used for predicting costs with these models. Finally, the procedure used to measure bias in the study will be explained and the "demographic variables" and the "condition variables" that were selected as likely candidates for being significant explainers of bias will be defined.

A. SELECTION OF THE SAMPLE

1. Data Sources

Data used in conducting the study were obtained from two sources: the U.S. Military Aircraft Cost Handbook [Ref. 22], and; the U.S. Missile Cost Handbook [Ref. 23]. Data contained in these handbooks were based on historical Service cost data and reflected the annual total obligational authority (TOA) flyaway costs for the included programs.

Aircraft data fell into five major categories: attack aircraft, fighter aircraft, bombers, attack helicopters, and patrol aircraft. Missile data fell into four main categories: air-to-air missiles, air-to-surface missiles, surface-to-air missiles, and surface-to-surface missiles. All cost data contained in these handbooks was normalized to a constant fiscal year base (FY-81). Consequently, the consistency of this data was ensured for all aircraft and missile programs.

Weapons systems contained in the aircraft handbook were U.S. Navy, Air Force, Marine Corps and Army aircraft. These aircraft were combat-oriented and were in the active U.S. inventory during the FY 1960-1980 period. Trainers, reconnaissance or electronic warfare variants and those aircraft produced for foreign military sales were not included in the handbook.

The missile handbook contained cost information for U.S. Air Force, Navy, and Army missile programs during the period FY 1961-1983.

Costs reflected in these handbooks were TOA dollars--the amounts budgeted in a specific fiscal year. TOA dollars do not reflect actual expenditures in any given fiscal year. Nevertheless, TOA dollars do provide an excellent proxy for actual dollar expenditures because it is customary within the DoD to ensure that expenditures match total obligational authority prior to the lapsing of an appropriation. As a

result, differences between TOA and actual expenditures are normally very small.

2. Selection Process and Criteria

Selection of aircraft and missile programs for inclusion in this study was based on three criteria: the number of plot points or fiscal years, the availability of airframe cost data and the completeness of the data. Only programs for which five or more fiscal years worth of data was available were included in the study. This criterion was required to ensure the minimum amount of data necessary for statistically fitting the cost progress models. The latter two criteria were established to ensure that meaningful analyses could be conducted. As a result, only aircraft programs which had complete quantity and airframe cost data were included in the study. In addition, only missile programs for which complete quantity, and guidance and control/airframe costs were available were included in the study.

3. Programs Selected for Study

Based on the three criteria described above, forty-six weapons procurement programs (fourteen missile programs and thirty-two aircraft programs) were selected for inclusion in this study. The specific aircraft and missile programs that comprised the sample are included in Tables 1 and 2 respectively.

TABLE 1

AIRCRAFT PROCUREMENT PROGRAMS INCLUDED IN THE STUDY

1.	A-3A/B	12.	AH-1S	23.	F-16A
2.	A-4C	13.	AV-8A	24.	F-101A/B/C
3.	A-4E	14.	B-47B/E	25.	F-102A
4.	A-4A/B	15.	F-3A/B/C	26.	F-105D
5.	A-4E/F	16.	F-4A	27.	F-105B/D
6.	A-6A	17.	F-4B	28.	F-111F
7.	A-6E	18.	F-4E	29.	P-2H
8.	A-7D	19.	F-4J	30.	P-3A
9.	A-7E	20.	F-4A/B	31.	P-3C
10.	A-37B	21.	F-8D/E	32.	S-3A
11.	AH-1G	22.	F-14A		

TABLE 2

MISSILE PROCUREMENT PROGRAMS INCLUDED IN THE STUDY

1. PHOENIX (AIM-54A)
2. SIDEWINDER (AIM-9D/G)
3. SIDEWINDER (AIM-9H)
4. SIDEWINDER (AIM-9H/L)
5. SPARROW (AIM-7E)
6. SPARROW (AIM-7F)
7. SHRIKE (AGM-45A)
8. STANDARD ER (RIM-67B/B-1/C-1)
9. STANDARD ER (RIM-67A)
10. STANDARD MR (RIM-66A)
11. STANDARD MR (RIM-66B)
12. TALOS (RIM-8E)
13. TARTAR (RIM-24B)
14. TERRIER (RIM-2E)

TABLE 2

B. COMPILATION OF PROGRAM DATA

Once the sample of aircraft and missile procurement programs had been selected, the following data were obtained for each program in the sample:

1. Program Name--name of the weapon system.

2. Manufacturer--name of the prime contractor.

3. Military Service--identified which Service branch(es) procured the weapon system.

4. Program Type--identified the nature of each weapon system, i.e., aircraft or missile.

5. Mission--identified each weapon system according to its primary mission. Aircraft program types were: fighter/attack, fighter, attack, bomber, patrol, and attack helicopter. Missile program types were: air-to-air, air-to-surface, surface-to-air, and surface-to-surface.

6. Modification--identified whether a particular weapon system was an entirely new design or a modification of an existing design.

7. Combined Program--identified whether a procurement program included several versions of a given weapon system or only one version.

8. Fiscal Year--identified the fiscal years during which quantities of a particular weapon system were procured.

9. Quantity--identified the number of units of each weapon system procured in a given fiscal year.

10. Aircraft Airframe Cost/Missile Guidance and Control Airframe Cost

C. EXPANSION OF THE SAMPLE

Following the compilation of data described in the previous section, the original data sample of forty-six programs was partitioned into 121 cost series. This "expansion of the sample" was accomplished by dividing each program cost series into individual, consecutive year-to-date cost series. For example, if a particular procurement program had cost data available for seven fiscal years, e.g., FY 1970-1976, this single cost series could be expanded into four separate cost series as shown below.

Cost Series # 1: FY 1970-1973 (used to predict 1974 cost)
Cost Series # 2: FY 1970-1974 (used to predict 1975 cost)
Cost Series # 3: FY 1970-1975 (used to predict 1976 cost)
Cost Series # 4: FY 1970-1976 (used to predict 1977 cost)

The initial cost series for each weapon system included in the sample was comprised of data from the first four fiscal years of that particular program (the minimum number of years needed to estimate the cost models). Each subsequent cost series for a given program was then created by additionally including the data point for the next fiscal year in the existing cost series. By partitioning the sample size in the study in this manner, it was possible to simulate actual usage of the

various cost progress models over time and evaluate their performance under varying data availability conditions.

D. SELECTION OF COST PROGRESS MODELS

Previous research has demonstrated that three factors are particularly useful in predicting the future costs of weapons systems: past cost, cumulative quantity, and production rate. Numerous models have been introduced in an effort to improve the quality of cost estimates over those obtained using the traditional learning curve model. These models are differentiated by the explanatory variables included and the underlying assumptions with regard to the relative importance of past cost, cumulative quantity and production rate in predicting future costs.

In addition, these models may be differentiated by the period or length of time over which data is observed and used in creating a forecast. Some models assume that future cost depends only on the most recent cost, quantity and/or production rate levels. However, other models assume that cost, quantity or production rate levels from early in a program's life are also significant and, as a result, specifically consider data covering the full production life. Regardless of which approach is used, there are both advantages and disadvantages associated with the use of data comprised of only recent observations or data comprised of both recent and earlier observations.

Use of recent observations may increase the relevance of model results in light of the current situation but sacrifices any information reflected in earlier, historical data. In addition, models based only on a few recent observations are more susceptible to random noise. Conversely, use of older observations may reduce the relevance of results or lead to results that are not representative of the current situation. However, the use of additional data reduces the impact of random variance in recent observations.

Finally, in addition to the two factors discussed above, cost progress models may be differentiated by the form of the assumed relationship between the dependant variable--cost--and the potential explanatory variables--past cost, cumulative quantity, and production rate. Relationships may be linear, log linear or some other form.

Four alternative models were selected for inclusion in this study: a random walk model, the traditional learning curve model, a model which expresses unit cost as a function of past cost and production rate (the fixed-variable model) and the common rate adjustment model.

1. Random Walk Model

The random walk model assumes that future cost is a function of past cost; however, only the most recent cost is relevant. Any deviation from predicted cost is considered random deviation.

The model is expressed as follows:

$$\hat{UC}_t = UC_{t-1}$$

where

$\hat{UC}_t$ = *Predicted unit cost in time period t.*
UC_{t-1} = *Actual unit cost in time period t - 1.*
t = *Time period.*

This model was selected for inclusion in the study for two reasons. First, it exemplifies the cost estimation method used by budget programmers when there is only very limited historical data. Under these circumstances, future cost projections are often based on actual costs in the previous period. Second, the random walk model is the most basic and naive cost estimation model and, as such, provides a useful benchmark for evaluating the performance of other more sophisticated models.

2. Traditional Learning Curve Model

The second model selected for inclusion in the study was the traditional learning curve. This model assumes that future cost is a function of both past cost and cumulative quantity produced. Moreover, all historical cost data is considered to be relevant.

The traditional learning curve model assumes a log linear relationship between cost and cumulative quantity and is expressed as follows:

$$\hat{UC}_t = aQ_t^b$$

where

$\hat{UC}_t$ = *Incremental unit cost of item at quantity Q.*
Q_t = *Cumulative quantity produced as of period t.*
a = *Theoretical first unit cost.*
$b = \frac{\log r}{\log 2}$ = *Learning curve exponent.*
r = *Learning rate.*

The traditional learning curve model was selected for inclusion in the study because it is the most widely researched cost progress model and represents the foundation on which other cost progress model variants are based.

3. Fixed-Variable Model

The third model selected for inclusion in the study was the fixed-variable model. This model specifically addresses the relationship between total unit cost and unit variable costs and unit fixed cost. Variable cost per unit remains constant; however, fixed cost per unit varies depending on the production volume because total fixed cost is allocated by spreading it over the total volume of output.

The fixed-variable model is expressed as follows:

$$\hat{UC}_t = a + b\left(\frac{1}{PR_t}\right)$$

where

$\hat{UC}_t$ = *Unit cost in period t.*
a = *Variable cost per unit.*
b = *Standard fixed cost per unit.*
PR_t = *Production rate* = $Q_t \div Q_{AVG}$.
Q_t = *Production quantity in period t.*
Q_{AVG} = *Average production quantity per period.*

This model was included in the study because it explicitly considers the impact of production rate on unit cost through the allocation of fixed overhead. In contrast to the traditional learning curve model (which includes cumulative quantity but not production rate as an explanatory variable), the fixed-variable model includes production rate but ignores cumulative quantity as an explanatory variable.

4. Bemis Production Rate Adjustment Model

The final model included in the study is the most widely used rate adjustment model. This model, popularized by Bemis [Ref. 16], was developed by augmenting the traditional learning curve model with a production rate term.

The model is expressed as follows:

$$\hat{UC}_t = aQ_t^b R_t^c$$

where

$\hat{UC}_t$ = *Predicted unit cost at quantity Q and production rate per period R.*
Q_t = *Cumulative quantity produced as of period t.*
R_t = *Production rate in period t.*
a = *Theoretical first unit cost.*
b = *Learning curve exponent.*
c = *Production rate exponent.*

The Bemis production rate adjustment model was included in the study because it considers both cumulative quantity and production rate (in addition to past cost). Hence, it is the most comprehensive of the four models in the study.

E. ANALYSIS OF BIAS

1. Unit Cost Prediction

In order to assess the bias exhibited by the random walk model, the traditional learning curve model, the fixed-variable model, and the Bemis model, predicted unit costs were estimated by applying each of the four models, in turn, to the actual cost series. The following paragraphs describe how this procedure was accomplished.

The random walk model assumes that unit cost in the next period is the same as unit cost in the current period.

Consequently, the predicted unit cost for any period (t) was simply the actual cost from the preceding period (t-1).

Predicted unit costs for the remaining three models were derived in the following manner. First, each of the models was separately fit to the initial cost series (comprised of the first four fiscal years' data) for each weapon system to derive the models' parameters. Then, the appropriate data values for cumulative quantity and/or production rate from period five were input into each of the models to obtain estimated unit costs for period five. These two steps were then repeated for each remaining cost series for the various weapons systems until predicted costs had been derived for every fiscal year for which actual cost data was available (e.g., models were next estimated on five years worth of data, then used to predict the cost for year six).

2. Bias Measurement-The Dependant Variable

Once predicted unit costs had been computed for each cost series with each model, a measure of bias was determined for each prediction as follows:

$$BIAS = \frac{PUC - AUC}{AUC}$$

where

BIAS = Percentage difference between predicted unit cost and actual unit cost.
PUC = Predicted unit cost from the particular model of interest.
AUC = Actual unit cost obtained from original unit cost data.

Positive BIAS values indicate a model has overestimated actual future cost; negative BIAS values indicate a model has underestimated actual future cost. BIAS values of zero indicate the predicted cost and actual future cost are identical and the associated model is unbiased.

As in the earlier study conducted by Moses [Ref. 5], BIAS represented the dependent variable in the statistical analysis. The basic objective of the study was to determine what factors or conditions are useful in explaining variance in BIAS.

The measures of bias and associated labels for each of the four models were as follows:

Bias for the Random Walk Model:	BIASRW
Bias for the Learning Curve Model:	BIASLC
Bias for the Fixed-Variable Model:	BIASFV
Bias for the Bemis Model:	BIASBE

Given 121 cost series (from the 46 procurement programs), there were 121 separate measures of bias for each model.

3. Explanatory Variable Selection

Model performance in prediction (i.e., the degree and direction of bias) depends on the circumstances in which the model is used. Two broad categories of factors which might influence model performance and, hence, be useful in explaining bias, were identified in the study. The first category consisted of "demographic variables" and the second category consisted of "condition variables". Together, these two groups of variables were the independent variables in the statistical analysis. Table 3 summarizes the independent variables and their corresponding labels.

TABLE 3

INDEPENDENT VARIABLES

Demographic Variables	Labels
Program Type	TYPE
Modification	MOD
Military Service	MILSERV
Mission	MISSION
Condition Variables	
Burden	BURDEN
Cost Variance	CVAR
Learning Rate	LRATE
Production Rate Variance	RATEVAR
Future Production Level	FUTUPROD
Past Production Trend	BEGTREND, ENDTREND
Plot Points	PLOTPNTS

a. Demographic Variables

The demographic variables describe characteristics of the weapons procurement program. Four demographic variables were considered in the study:

1. Program Type
2. Modification
3. Military Service
4. Mission

These particular demographic variables were selected for investigation because they represent readily apparent characteristics of the various programs which may affect model bias.

b. Condition Variables

In addition to the four demographic variables, seven condition variables were considered in the study:

1. Burden
2. Cost Variance
3. Learning Rate
4. Production Rate Variance
5. Future Production Level
6. Past Production Trend
7. Plot Points

The label "condition variables" is used here because each of these variables in some manner indicates something about conditions that existed during the weapon system procurement. These are analogous independent variables (factors) to those examined by Moses [Ref. 5]. The following paragraphs describe each of these variables along with the underlying rationale for their inclusion in the study.

(1) Burden

Burden (BURDEN) indicates the percentage of total unit cost made up of fixed cost. Burden is measured as follows:

$$BURDEN = \frac{b}{a + b}$$

where

a = Variable cost per unit (constant).
b = Standard fixed cost per unit.

Note: a and b were estimated parameters from the fixed-variable model.

Burden was included in the study because past research (Linder and Wilbourn [Ref. 10], Moses [Ref. 5] and Smunt [Ref. 24]) has shown that burden directly influences the impact of changes in production rate on unit cost. As production rate increases, the cumulative quantity produced during a period increases and the variable cost per unit decreases due to the

incidence of learning. In addition, as production rate in a period increases, fixed cost per unit is reduced as total fixed cost is spread over a larger production output. The impact of production rate increases as the proportion of total cost made up of fixed costs increases. As a result, the relative bias of the various models may depend on the level of burden.

(2) Cost Variance

Cost variance (CVAR) indicates the amount of unsystematic variation in unit cost that may result from unanticipated, non-recurring, random factors. Examples include changes in the cost, type or availability of input resources, temporary fluctuations in efficiency, and unplanned changes in the production process.

By assessing the amount of unsystematic variation associated with various weapons procurement programs, it may be possible to determine whether there is a relationship between cost stability and bias. For this reason, cost variance was examined in the study.

Cost variance was measured as follows:

$$CVAR = \frac{\frac{\sum |C_t - C_{AVG}|}{C_{AVG}}}{n}$$

where

$CVAR$ = *Cost variance.*
C_t = *Unit cost for a given production period.*
C_{AVG} = *Average unit cost for all production periods to date.*
n = *Total number of production periods.*

(3) Learning Rate

Learning rate (LRATE) measures the decrease in per unit cost that occurs as the quantity of units manufactured doubles. It is affected by the type of production process and the complexity of the product design. Smunt [Ref. 24] found that the degree of learning in the underlying production process determines the improvement in prediction accuracy that results from including a learning parameter in a model. Learning rate was examined in the study to determine the nature of the relationship between learning rate and bias for the various models.

Learning rate was measured as follows:

$$r = 2^b$$

where

r = Learning rate expressed as a percentage.
b = Learning curve exponent as estimated by the traditional learning curve model.

(4) Production Rate Variance

Production rate variance (RATEVAR) reflects the severity of period-to-period fluctuations in production rate. These fluctuations may result from either changes in demand for a particular weapon system or changes in the cost or availability of production inputs. Production rate variance was measured as follows:

$$RATEVAR = \frac{\frac{\sum |Q_t - Q_{AVG}|}{Q_{AVG}}}{n}$$

where

RATEVAR = Rate variance.
Q_t = Quantity of units produced in the current period.
Q_{AVG} = Average quantity of units produced for all periods to date.
n = Total number of periods to date.

(5) Future Production Level

Future production level (FUTUPROD) indicates whether the level of production in the next period (the period for which cost is to be forecast using the model) is high or low relative to the current period. Future unit costs are predicted using cost progress models fit to past production data. As a result, the accuracy of the various models may be affected if the production level in the period for which costs are being estimated varies significantly from the production levels that existed during previous periods. Future production level was examined in the study to determine whether production growth and production cutbacks affect the tendencies of the various models to over/underestimate unit costs.

Future production level was measured as follows:

$$FUTUPROD = \log \left(\frac{Q_N}{Q_L}\right)$$

where

$FUTUPROD$ = *Future production level.*
Q_N = *Production level for the next period for which costs are being forecast.*
Q_L = *Production level for the last (most recent) period.*

(6) Past Production Trend

Past production trend indicates the pattern of production volume associated with each weapon system included in the study. Two variables, BEGTREND and ENDTREND, were used to reflect how production volume was changing at the beginning and end of each cost series to which models were fit. Production rate per period may be low initially in order to work out bugs and ensure a stable product design prior to full scale production. Alternatively, initial production rate per period may be high if the current weapon system represents an updated version of an already existing system with only minor modifications.

Production rate per period may also vary at the end of the production run depending on whether the program is abruptly cancelled, gradually phased out or continued at some minimum level in order to maintain a warm production base. These two production trend variables were included in the study to determine whether the bias associated with the various models was related to past production trend.

The production trend variables were measured as follows:

$$BEGTREND = \frac{Q_{AVG} - Q_1}{Q_{AVG}}$$

and

$$ENDTREND = \frac{Q_L - Q_{AVG}}{Q_{AVG}}$$

where

$BEGTREND$ = *Production trend at the beginning of the production run.*
$ENDTREND$ = *Production trend for the most recent production period.*
Q_1 = *Quantity produced in first production period.*
Q_L = *Quantity produced in most recent period.*
Q_{AVG} = *Average quantity per period produced through current period.*

Positive values for BEGTREND and ENDTREND indicate production trends that are increasing in volume.

(7) Plot Points

The final independent variable included in the study was the number of plot points (PLOTPNTS). Plot points indicates the number of data points available to estimate model parameters. The accuracy of the learning curve, fixed-variable, and Bemis models should improve as the amount of data available during the model estimation period increases. Nevertheless, if a model is inherently biased,

increasing the number of plot points will not necessarily eliminate the bias associated with the model. The number of plot points was considered in the study to determine whether or not the bias associated with the various models could be explained in terms of the availability of past production data.

F. SUMMARY

This chapter has provided an overview of the sample and variables to be used to investigate bias. The discussion began with a description of the sample selection process, and a summary of the data collected for each program included in the sample. This was followed by a description of the procedure used to expand the sample from 46 observations (programs) to 121 cost series. Next, the four cost progress models included in the study--the random walk model, the traditional learning curve model, the fixed-variable model, and the Bemis production rate adjustment model were described along with the rationale for their selection. The cost estimation procedure was then explained and a method for measuring bias was introduced. Finally, two categories of independent variables--demographic variables and condition variables--were introduced to be evaluated as potential sources of model bias.

IV. ANALYSIS AND FINDINGS

Chapter III introduced eleven variables--four demographic variables and seven condition variables--which might influence the performance of alternative cost progress models and, hence, be useful in explaining the cost prediction bias of these models. This chapter describes the statistical procedures that were performed to assess the significance of each of these variables in explaining bias. It will begin by providing an overview of the statistical tests that were used to analyze the variables. This will be followed by a presentation of some general findings with respect to the performance of the four alternative cost progress models. Next, model specific findings regarding the significance of each of the eleven explanatory variables will be presented. Finally, the results of the analysis will be summarized and conclusions will be presented.

A. STATISTICAL PROCEDURES

Prior to conducting statistical analyses, the distribution of values for each of the dependent and independent variables was assessed. The presence of extreme values within the data set could unduly influence the outcomes of the statistical analyses. Accordingly, variable values which lay beyond three standard deviations from the mean for the variable were

truncated. Truncation involved replacing extreme values with values equal to three times the standard deviation for the appropriate variable. This approach was applied because it reduces the influence of outliers on the results of the statistical analyses without discarding and, consequently, ignoring the impact of these observations.

Once all extreme values had been identified and truncated, the dependency of BIAS on the demographic variables was evaluated for each model using analysis of variance (ANOVA). After determining the significance of the demographic variables in explaining BIAS, the significance of the seven condition variables and MOD was assessed using both simple and multiple linear regression analysis. Finally, three different sets of Spearman and Pearson correlation analyses were conducted. First, correlation coefficients were computed for the measures of BIAS from the four different models. Next, correlations between the condition variables (including MOD) and BIAS were examined for each model. Then values for the condition variables (including MOD) were correlated with each other in an effort to detect potential multicollinearity.

In conducting tests of statistical significance, findings with alpha values less than 0.01 were considered significant. When analyzing pairwise correlations between the explanatory variables, correlation coefficients larger than 0.50 were regarded as offering strong evidence that multicollinearity might be a problem.

B. ANOVA RESULTS

Analysis of variance was performed for each of the four cost progress models to determine whether or not prediction bias could be explained by any of the demographic variables. The following demographic variables were examined:

1. TYPE
2. MILSERV
3. MISSION
4. MOD
5. TYPE x MOD (interaction variable)

None of the demographic variables tested were significant in explaining the bias of cost predictions made with the random walk, fixed-variable or Bemis models. However, MISSION, MOD and the TYPE x MOD interaction variable were all found to be significant in their ability to explain variations in the bias of learning curve cost predictions. ANOVA results for the learning curve model are provided in Table 4. Findings concerning the significance of MOD in explaining variations in the bias of learning curve cost predictions will be discussed later, along with the regression results. Differences in learning curve bias due to MISSION and TYPE are simply noted. No hypothesis was offered to expect differences in learning curve cost prediction bias in relation to MISSION and TYPE.

TABLE 4

LEARNING CURVE MODEL COST PREDICTION BIAS ANALYSIS OF VARIANCE RESULTS

ANALYSIS OF VARIANCE PROCEDURE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F
MODEL	10	6.17667664	0.61766766	5.84	0.0001
ERROR	110	11.64039114	0.10582174		
CORRECTED TOTAL	120	17.81706778			

R-SQUARE	C.V.	ROOT MSE		BIASLC MEAN
0.346672	-3913.238	0.32530253		-0.00831287

SOURCE	DF	ANOVA SS	MEAN SQUARE	F VALUE	PR > F	
TYPE	1	0.59489073	0.59489073	5.62	0.0195	
MILSERV	3	0.20565072	0.06855024	0.65	0.5860	
MISSION	4	2.53769578	0.63442394	6.00	0.0002	*
MOD	1	1.29598087	1.29598087	12.25	0.0007	*
TYPE*MOD	1	1.54245854	1.54245854	14.58	0.0002	*

* Indicates $\alpha < 0.01$

Consequently, findings related to these factors were not pursued further but are mentioned here for possible investigation in future research. Based on the fact that the utility of these variables in explaining model bias was limited to the learning curve model, the remainder of the study was devoted to examining the significance of the condition variables (including MOD) in explaining model bias.

C. OVERALL MODEL PERFORMANCE

1. Mean Bias

Summary results for the average level of bias associated with cost predictions made using the random walk, learning curve, fixed-variable and Bemis models are provided in Table 5. The mean bias of predicted unit costs estimated with the four models ranged from -0.008313 for the learning curve to 0.375045 for the fixed-variable model. The results show that on average, the learning curve underestimated unit costs by approximately 0.83% while the Bemis, random walk and fixed-variable models overestimated unit costs by approximately 2.9%, 4.6% and 37.5% respectively. Bias measures for all four models were skewed in the positive direction. Hence, measures of the median bias were examined in an effort to obtain a more objective assessment of model performance.

TABLE 5

RANK ORDERING OF MODELS BY MAGNITUDE
OF POSITIVE BIAS (Lowest to Highest)

Model	Mean Bias	Median Bias	Standard Deviation	Skewness
Learning Curve	-0.008313	-0.060802	0.385325	3.708457
Bemis	0.029472	-0.012925	0.339456	0.838916
Random Walk	0.045842	0.015789	0.199953	1.396772
Fixed-Variable	0.375045	0.188371	0.929000	3.391860

2. Median Bias

The median bias of predicted unit costs estimated with the four models ranged from -0.060802 for the learning curve to 0.188371 for the fixed-variable model. When the median bias measurements for the various models were compared, the relative ordering of the models according to magnitude of positive bias remained the same. However, the magnitude of positive bias decreased significantly in all cases. This suggests that some observations had a very large positive bias, causing the mean bias to be more positive than the median. The direction of bias remained unchanged for all models except for the Bemis model. In this case, mean model bias was approximately 2.9% while median model bias was approximately -1.3%. These results seemed to indicate that the learning curve and Bemis models underestimate unit cost (provide low unit cost estimates) while the random walk model and especially the fixed-variable model overestimate unit cost (provide high unit cost estimates). Perhaps not coincidentally, both the learning curve model and the Bemis model use cumulative quantity to predict future cost, while neither the random walk model nor the fixed-variable model contain a cumulative quantity term.

3. Model Bias Correlations

Both Pearson correlation coefficients and Spearman correlation coefficients were computed for the values of bias associated with each of the four cost progress models. The results of these correlation analyses are provided in Table 6. All Spearman correlation coefficients were positive and significant; correlation values ranged from 0.27220 to 0.58048. However, these results were not completely confirmed by the computed Pearson correlation coefficients. Values obtained from the Pearson correlation analysis indicated significant bias correlations for only three of the six paired model combinations: BIASRW-BIASLC, BIASRW-BIASBE, and BIASBE-BIASFV. Significant Pearson correlation coefficients ranged from 0.26401 to .063335. Both correlation analyses indicated that the strongest positive correlation existed between random walk model bias and learning curve model bias. The second highest correlation in both analyses existed between random walk model bias and Bemis model bias. The third relationship which was correlated and significant in both analyses was Bemis model bias and fixed-variable model bias.

Based on the outcomes from the correlation analyses, it was concluded that there is evidence of a positive correlation between bias for all paired model combinations.

Table 6

CORRELATION ANALYSES OF MODEL PREDICTION BIAS

Spearman Correlation Coefficients

	BIASRW	BIASLC	BIASFV	BIASBE
BIASRW	1.00000 0.0	0.58048* 0.0001	0.27220* 0.0025	0.34698* 0.0001
BIASLC	**0.63335*** **0.0001**	1.00000 0.0	0.35518* 0.0001	0.43751* 0.0001
BIASFV	**-0.02073** **0.8215**	**-0.04602** **0.6162**	1.00000 0.0	0.54488* 0.0001
BIASBE	**0.26401*** **0.0034**	**0.13296** **0.1460**	**0.35905*** **0.0001**	1.00000 0.0

Pearson Correlation Coefficients

Correlation Coefficients/Prob>|R| under H_o: *Rho*=0

* *Indicates* $\alpha \leq 0.01$

That being the case, it seemed reasonable to suspect that, to the extent prediction bias was positively correlated among the various models, the random walk, learning curve, fixed-variable and Bemis models might perform in a similar manner under the same circumstances. Moreover, it was possible that the significance of various explanatory variables in explaining prediction bias might be similar between models whose prediction bias was highly correlated. In order to resolve these issues, correlation analyses and regression analyses were used to study the relationship between model bias and the explanatory variables. The following paragraphs provide the results of these analyses for each of the four models.

D. REGRESSION ANALYSES AND RESULTS

The significance of the seven condition variables and one demographic variable--MOD--was assessed using regression analysis. First, simple regression analyses were performed to independently test the significance of these independent variables in explaining prediction bias when considered in isolation. Then multiple regression analyses were conducted to determine the significance of these same independent variables in explaining prediction bias while controlling for the effects of the other independent variables. Pairwise correlations were additionally determined among both independent and dependent variables. The following sections

describe the results of these analyses for each of the four models. The discussion of the results will be organized by model type in the following sections. Summary tables containing average bias for particular subsets of the sample (Table 7) and correlations (Table 8) are provided here. They will be referred to as the discussion proceeds. Regression results will be presented in each section that follows.

1. Random Walk Model Bias (BIASRW)

Table 9 provides the results of the multiple regression analysis for BIASRW. The results indicate that approximately 33% of the variation in random walk model bias was explained by the nine independent variables. However, only two variables--LRATE and FUTUPROD--were significant in explaining variations in the bias of random walk model cost predictions. The relationships between random walk model bias and each of these variables are depicted in Figure 1.

TABLE 7

RELATIONSHIP BETWEEN MODEL BIAS AND
SIGNIFICANT EXPLANATORY VARIABLES

MODEL/ INDEPENDENT VARIABLE	MEAN BIAS FOR EACH LEVEL		
	LOW[4]	MEDIUM[5]	HIGH[6]
RANDOM WALK:			
LRATE	0.052212	0.027690	0.074813
FUTUPROD	-0.064090	0.024403	0.199371
LEARNING CURVE:			
BURDEN	0.024573	-0.082520	0.111077
LRATE	-0.151880	-0.054310	0.219654
BEGTREND	0.194725	-0.056370	-0.122000
ENDTREND	-0.039350	-0.013680	0.032102
FUTUPROD	-0.170490	-0.048520	0.235615
FIXED-VARIABLE			
FUTUPROD	0.907445	0.267981	0.060342

[4] Identifies variable values from the first quartile of the variable's distribution.

[5] Identifies variable values from the second and third quartiles of the variable's distribution.

[6] Identifies variable values from the fourth quartile of the variable's distribution.

TABLE 8

CORRELATION ANALYSIS OF INDEPENDENT VARIABLES AND MODEL BIAS

SPEARMAN CORRELATION COEFFICIENTS

VARIABLES	MODEL BIAS			
	BIASRW	BIASLC	BIASFV	BIASBE
PLOTPNTS	-0.17974	-0.05108	0.10070	0.01313
MOD	0.07265	**0.32932***	0.00535	0.15803
BURDEN	0.03855	-0.15101	-0.03751	-0.08670
CVAR	0.09104	**-0.31316***	0.13307	-0.13021
LRATE	-0.07992	**0.34364***	**-0.25810***	0.07666
RATEVAR	0.10188	-0.06053	0.02671	-0.01795
BEGTREND	0.00121	**-0.28616***	**0.27543***	0.05199
ENDTREND	0.13639	-0.02809	0.10935	0.00670
FUTUPROD	**0.42949***	0.34583	**-0.28896***	-0.11105

PEARSON CORRELATION COEFFICIENTS

VARIABLES	MODEL BIAS			
	BIASRW	BIASLC	BIASFV	BIASBE
PLOTPNTS	-0.18426	-0.10901	0.08183	-0.04182
MOD	0.10132	**0.26970***	-0.03685	0.14173
BURDEN	0.21469	**0.29738***	0.02674	-0.07563
CVAR	0.03327	-0.09775	-0.02379	-0.11590
LRATE	0.21866	**0.68339***	-0.20121	-0.01375
RATEVAR	0.04942	-0.02878	-0.13466	-0.12115
BEGTREND	-0.09221	**-0.35540***	0.13413	0.02868
ENDTREND	0.08477	0.07822	0.03699	-0.02651
FUTUPROD	**0.46612***	**0.31543***	**-0.57893***	-0.18493

* *Indicates* $\alpha \leq 0.01$

TABLE 9

MULTIVARIATE ANALYSIS OF RANDOM WALK MODEL BIAS

Analysis of Variance

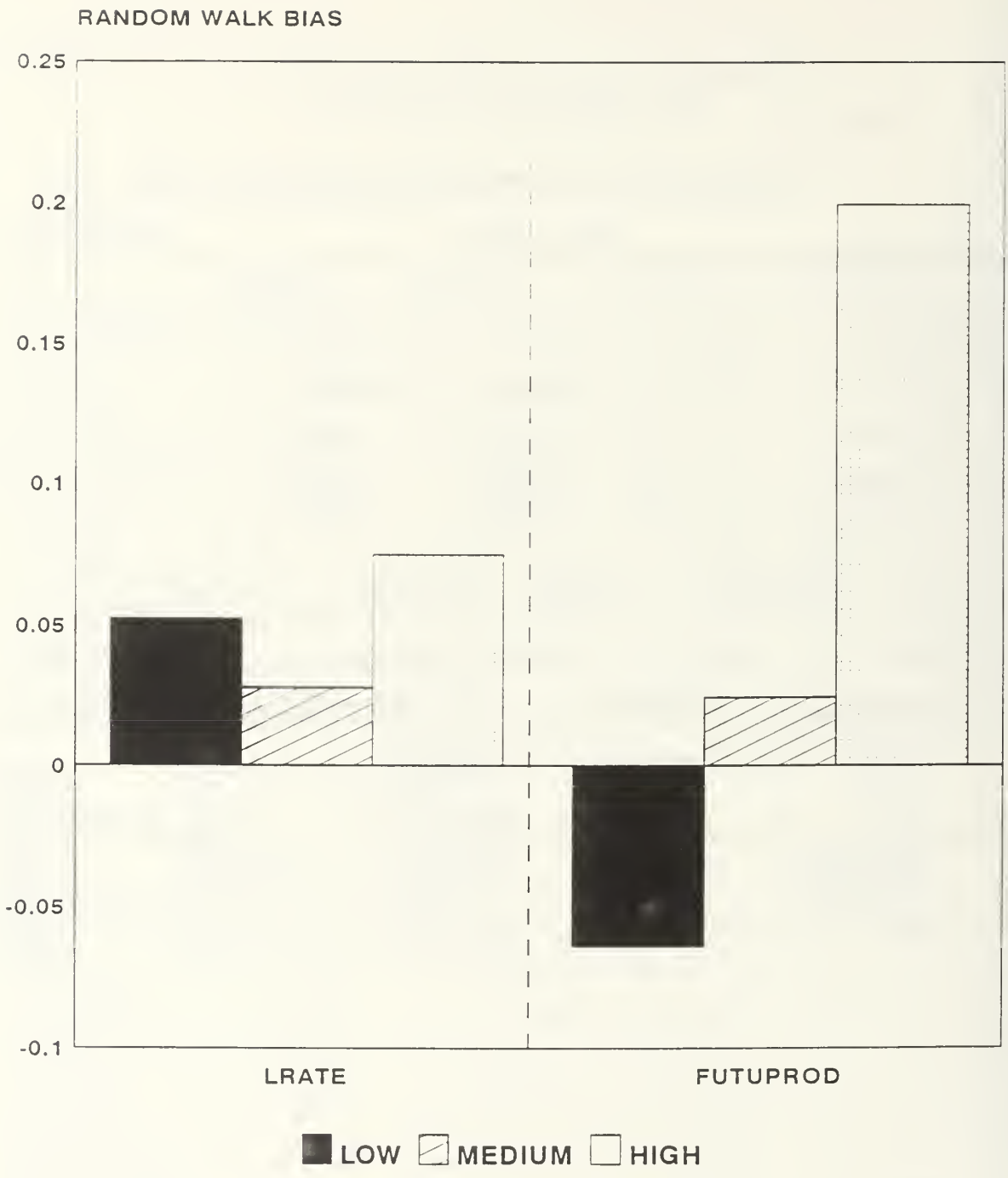
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	9	1.62871	0.18097	6.709	0.0001
Error	96	2.58963	0.02698		
C Total	105	4.21833			

Root MSE	0.16424	R-square	0.3861
Dep Mean	0.05042	Adj R-sq	0.3285
C.V.	325.76890		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|	
INTERCEP	1	-0.366140	0.15950058	-2.296	0.0239	
BURDEN	1	0.159379	0.06505715	2.450	0.0161	
CVAR	1	-0.034241	0.14791824	-0.231	0.8174	
LRATE	1	0.447784	0.15651890	2.861	0.0052	*
RATEVAR	1	-0.062376	0.12325481	-0.506	0.6140	
BEGTREND	1	0.136147	0.06080772	2.239	0.0275	
ENDTREND	1	0.044053	0.02678170	1.645	0.1033	
FUTUPROD	1	0.126599	0.02233301	5.669	0.0001	*
PLOTPNTS	1	-0.016620	0.00932812	-1.782	0.0780	
MOD	1	0.048861	0.04634494	1.054	0.2944	

* *Indicates* $\alpha \leq 0.01$



Relationship Between Random Walk Model Bias and Significant Explanatory Variables

Figure 1

a. LRATE

LRATE was found to be significant in explaining variations in the bias of random walk model cost predictions in both the simple and multiple regression analyses. The estimated regression coefficients indicated a positive relationship between LRATE and BIASRW. Computed Spearman and Pearson correlations were inconclusive with regard to the nature of the relationship. Figure 1 indicates that the random walk model bias was positive for all levels of LRATE. However, the relationship between LRATE and BIASRW appears counterintuitive. One would expect low (high) values of LRATE to be associated with high (low) values of bias. In fact, a nonlinear relationship exists. Bias is highest (most positive) when the learning rate is at extremes, i.e., when the learning rate is steepest or most shallow. Bias is lowest when the learning rate is in the middle range, i.e., where most programs likely will fall.

b. FUTUPROD

In addition to LRATE, FUTUPROD was also found to be significant in explaining random walk bias in both the simple and multiple regression analyses. The estimated regression coefficient was positive in both analyses thus indicating a positive relationship between FUTUPROD and BIASRW. Computed Spearman and Pearson correlation coefficients for FUTUPROD and BIASRW were both moderately positive and significant, thereby

confirming this relationship. The results show that when FUTUPROD was low (high) the level of positive bias was low (high). This relationship is readily apparent in Figure 1 which shows clearly that when FUTUPROD was low (high), the random walk model underestimated (overestimated) unit costs. These results were expected. When FUTUPROD is high (low), unit costs are normally lower (higher) in the period being forecast because of decreasing (increasing) variable costs per unit (learning effect) and allocation of total fixed costs over a larger production volume.

2. Learning Curve Model Bias (BIASLC)

Table 10 provides the results of the multiple regression analysis for BIASLC. The results indicate that approximately 73% of the variation in learning curve prediction bias was explained by the nine independent variables. Five of the independent variables included in the multiple regression analysis were significant in explaining variations in the level of bias of learning curve model cost predictions. These variables were: BURDEN, LRATE, BEGTREND, ENDTREND, and FUTUPROD. In general, the simple regression results agreed with the multiple regression results in terms of the significance of these variables.

TABLE 10

MULTIVARIATE ANALYSIS OF LEARNING CURVE MODEL BIAS

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	9	12.92818	1.43646	32.090	0.0001
Error	96	4.29728	0.04476		
C Total	105	17.22546			

Root MSE	0.21157	R-square	0.7505
Dep Mean	-0.00776	Adj R-sq	0.7271
C.V.	-2727.49365		

Parameter Estimates

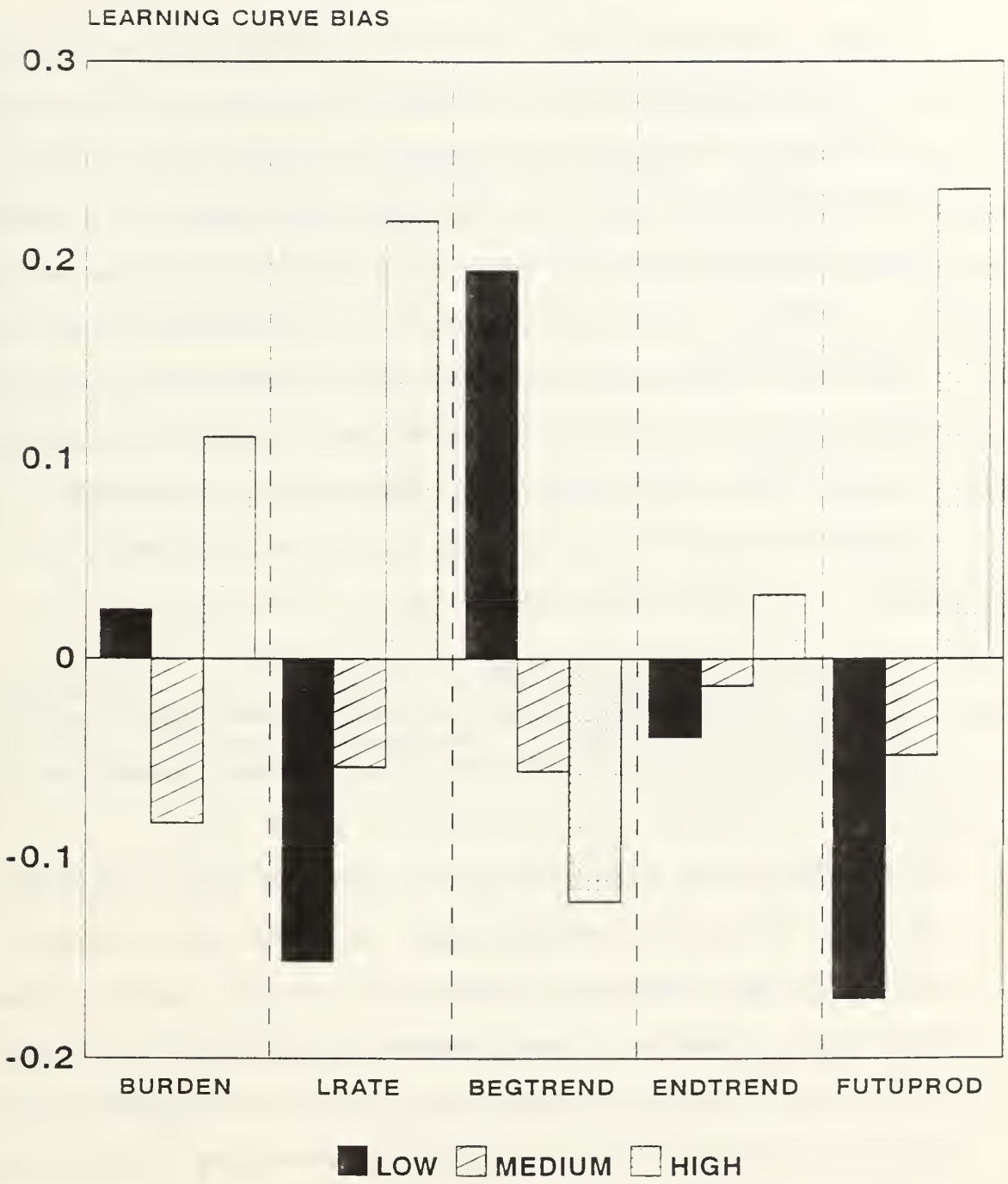
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|	
INTERCEP	1	-2.325371	0.20546621	-11.318	0.0001	
BURDEN	1	0.372917	0.08380563	4.450	0.0001	*
CVAR	1	0.127763	0.19054602	0.671	0.5041	
LRATE	1	2.316325	0.20162526	11.488	0.0001	*
RATEVAR	1	-0.204656	0.15877497	-1.289	0.2005	
BEGTREND	1	0.283523	0.07833157	3.620	0.0005	*
ENDTREND	1	0.175378	0.03449978	5.083	0.0001	*
FUTUPROD	1	0.122591	0.02876904	4.261	0.0001	*
PLOTPNTS	1	-0.011255	0.01201634	-0.937	0.3513	
MOD	1	0.137628	0.05970084	2.305	0.0233	

* *Indicates* $\alpha \leq 0.01$

However, there were two noteworthy differences: (1) ENDTREND was significant in the multiple regression analysis but was not significant in the simple regression analysis; and (2) MOD was significant in the simple regression analysis but was not significant in the multiple regression analysis. These two findings will be discussed in detail later in this section. The following paragraphs discuss the results of the regression analyses for each of the variables identified as being significant. The relationships between the level of each of the significant independent variables and model bias are depicted in Figure 2.

a. BURDEN

BURDEN was found to be significant in explaining variations in model bias in both the multiple and simple regression analyses. The estimated multiple regression coefficient indicates there was a positive relationship between burden and prediction bias for the learning curve. This result was confirmed by the significant but relatively weak, positive Pearson correlation (See Table 8). These findings indicate that when the proportion of total cost made up of fixed costs was high (low), the level of positive bias was high (low). Figure 2 graphically depicts the relationship between learning curve model bias and BURDEN. The graph shows that low (high) positive bias was indeed associated with low (high) levels of BURDEN.



Relationship Between Learning Curve Model Bias and Significant Explanatory Variables

Figure 2

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Thesis
T121 Tagg
c.1 Evaluating the bias of alternative cost progress models.

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